# NOTES ON THE ART OF SOLVING PROBLEMS

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These notes are pointed at the kinds of problems students encounter in courses in science, but most of the ideas apply to people's reaching all kinds of goals they have chosen for themselves -- i.e., to solving problems in general. There is an enormous literature on this kind of activity~ here is a brief distillation of it.

Problem solving ability is, in part, the possession of technical skills (e.g., previous experience, background knowledge, training, education.). But a more unappreciated and perhaps more important part is the development of esthetic responses that cause one to appreciate and follow directions of thought that lead to solutions of problems. In this combination of technical and esthetic factors, becoming a better problem-solver is like becoming a better artist, musician, or writer.

In practicing, get away from the tyranny of the clock, of time, of feeling the need to accomplish a certain tangible amount by a certain time. Instead, recognize that moderate amounts of good, unhurried practice, done right, will lead over the months to more growth than rushed efforts that "solve" lots of problems.

Nurture your own self-confidence~ don't berate yourself for not being perfect~ appreciate what you CAN do. Acknowledge that there will always be problems you cannot solve~ that other people will solve problems better than you. Don't worry about other people's successes. Work toward your own growth, not your own perfection.

Take responsibility for your own actions or inactions~ don't try to cop out from the need to work to attain valuable long-term growth. If you choose not to work hard enough to develop skills, simply accept that the consequences will be a lack of those skills and whatever else might flow from that lack. Conversely, if you do work hard enough to develop those skills, the result is not a better person, but the same person with skills. Lack of work, errors, or even attacks by others on your statements or products are not faults in, or attacks on, your self. They relate only to your performance and you are not your performance. If you're not regularly making errors, in public, you're probably playing it too cautiously and not optimizing growth and enjoyment in life. Errors, not dismissed but carefully analyzed, offer dramatic and thus memorable 'experiences to profit from, tailored specifically to you.

Forcibly hold your attention on the problem at hand, even in the face of distractions. Don't switch attention to distractions. Just acknowledge that the distractions are there and you are going to do your work anyway. If you habitually work in distracting environments, look at your motivations. Do you really want to concentrate and solve problems or not?

Most worthwhile problems contain far more elements than can be handled by the short-term span of the human memory (This span is some 4 to 8 elements.). The subconscious works on problems at any and all times, so profit by seemingly idle periods in your efforts; interrupt heavy concentration with routine tasks. You might be surprised at the ideas you get while brushing your teeth!

The following are discussions of the main steps in solving a problem:

## 1. ACCEPT THE PROBLEM

If you want to solve a problem, don't spend your time and emotional energy fighting it; spend your time solving it instead. That sounds obvious, but it is commonplace for people to agonize enormously about how bad their problems are, how unclear they are, how unfair they are. Yours may indeed be poorly worded, hard, trivial, boring, unrealistic, unfair. Most problems are. That doesn't make them go away. Most problems in the real world resemble classroom problems in these respects. The real world is unfair -- who said it was fair? Notice whenever you start to fight your problem and take some time off for a punching bag or karate practice. If you hear yourself complaining that your problem is "too ...", that will be a clue that you're fighting it. Your problem may indeed be ..., it may be very..., but YOU added the word "too." The word "too" simply means that it is more ... than you like. Tough. Presumably, you had some motivation to get there with that problem. Focus on your desires, your real motivations, for approaching the problem in the first place. Fighting problems is people's biggest barrier to solving them.

## 2. UNDERSTAND THE PROBLEM

Understanding the problem is often more than half the battle; don't rush through this understanding process. In understanding it and making it your own, focus on these three questions:

1. Exactly what situation or condition is the problem dealing with?

2. What data or facts or set of constraints are known that make the situation special?

3. What is called for or desired?

People often rush through their understanding of the problem in their haste to begin solving something. Don't omit things that are in the problem; don't read in material that isn't there. This improves with practice. Expert problem-solvers in physics and chemistry approach their problems first using <u>vague English-language sentences and sentence fragments.</u> They work at a very low level of detail and recall. In order to get from A to Z, for example, they know there is a way to go from A to Band from B to C and from C to D, and since they know there is a way to go from Z to D, they could use that path backward to get from D to Z, and thus in principle they see their path from A to Z. Their problem is solved in principle, though they have not recalled any of the detailed steps.

That comes later when it is appropriate. They use quick drawings in attempts to catch the essence of the problem. They scribble down all the given information in THEIR OWN terms so they can put away the book from which the problem comes. Thus, the problem posed becomes THEIRS, rather than remaining that of the author who wrote it.
Is the problem consistent with your values? With your highest values? If not, can you renegotiate those aspects that conflict with your values? After all, you are doing all this to create a rich, satisfying set of life patterns that will serve you well throughout your life, and in the process serve others. Don't get caught up in simply being a "brain for hire." Failure to understand the problem thoroughly is people's next biggest barrier to solving it, after fighting it.

### 3. ANALYZE THE PROBLEM

Put its features into YOUR OWN theoretical terms. These may be mathematical symbols or forms, or the words from which these will flow. What are the special conditions of your problem (e.g., things that stay constant throughout the process)? What qualitative predictions can you make about the answer? Retrieve relevant basic information from your memory or notes or books. Get a USEFUL formulation of the problem.

### 4. DESIGN THE SOLUTION

If the problem is not immediately solvable, you will probably have to decompose it into a series of sub problems, each of which is solvable. In principle, you can keep decomposing this way until you've charted a path to the answer, or from the answer back to the original data of the problem. To do this requires a knowledge base of basic solvable problems, which are BUILDING BLOCKS for problem solving. To make something like $pV = nRT$ into a building block for a whole set of solvable problems requires ALL the following skills:

1. Knowing that you know it and having the ability to retrieve it.

2. Ability to transform it (e.g., $T = pV/nR = pVM/mR$).

3. Ability to re-describe it, i.e., change the symbolic form among words, symbols, graphs, etc.

4. Ability to apply it, i.e., correctly attach quantities to the things the symbols represent, and get the right answer.

Keep a list of the various building blocks you have as tools. This is like a crib-sheet for an exam, only keeping it and reviewing it are important tools for your expanding self-confident problem solving.

In planning to solve the problem, focus on how the unknown or desired result is connected to the starting point, the data. If thin connection is not immediately clear, what

auxiliary problems might develop a connection?

Have you seen the problem before? Have you seen it in a slightly different form? Do you know a related problem? Do you know a relationship or equation which might be useful? Look at the unknown and try to think of a familiar problem having the same or similar unknown. Hunt for a problem related to yours that has been solved by someone else before. Can you use it? Its result? Its method? Could you introduce some auxiliary element in order to make its use possible?

Go back and restate the problem. Restate it still differently. Go back to the definitions of the words and concepts used in the problem and restate it again in terms of these definitions, instead of the words and concepts themselves. Anything to break out of the rut you have established.

Are you reading constraints into the conditions of the problem that need not be there? If so, lift them.

See what you CAN get from the material at hand even if it doesn't seem to go in the direction of a solution. Can you derive anything at all of potential use from the data? Can you solve any part of the problem? Work backward from a possible solution so see if you can hook it up with the problem itself.

DESIRE the solution. Follow your emotions. Expect to succeed. Enjoy the process.

**You will make mistakes; accept them.** Make them overtly. Learn from them.

Don't get overly committed to a single method of approach: withhold judgment on whether a method will work until it proves it will. Ask a variety of questions about what is going on in your problem. Do you consider questions to be something asked only by OTHER people? Poke at your problem.

Communicate, explain to others the problem and how you are trying to solve it, even if the solution is not clear. Often just stating the problem to someone else is enough to create the 8olution. If no one else is available, explain it out loud to some imaginary (and not too bright) person.

Have you used all the data? You might not have to, of course. Have you taken into account all essential notions involved in the problem?

## 5. FIND TENTATIVE SOLUTION

Only at this stage should you solve in detail the various sub problems. Only here does the math enter. Substitute specific numbers (WITH UNITS!) only at the last minute. Get a tentative solution. Be sure to use the units of each mathematical quantity, and manipulate them as if they were numbers, canceling them when appropriate, and letting the units of

the answer appear naturally as what results from the process. Check each step.

I find that I make fewer mistakes and do a better job if I write out my problem solutions THE FIRST TIME in ink, not in pencil, and rather neatly and carefully. Ink has the effect of committing me to what I write, even though I often cross out terms, lines or whole paragraphs or pages. The use of the sloppy, pencil scratch-copies that so many people employ is not a good habit.

## 6. ASSESS, REVISE, AND APPRECIATE

Examine your solution! Don't just draw a box around it, heave a sigh of relief, and pass on to the next problem. Problems in the real world often have FAR more serious consequences than homework problems in college. So take your answers seriously. Can you check the result? Can you check the argument used to get the result? If possible, derive the result by a different method. Repetition of exactly the same procedure is a poor way to catch mistakes, because people follow their previous channel. See if the result actually solves the problem. Check out limiting or special cases of your solution. DOES YOUR RESULT MAKE COMMON SENSE? Challenge your solution. Try to show it to be wrong. Can you use the result or the method for some other problems? Is the sign right? Is the answer of the right magnitude? Do the units make sense?

Let your problem and its solution increase your intuition. Think back over your experience of solving it and over your results, and incorporate everything learned into your appreciation of reality, so that in the future, your feeling of what "makes sense" w ill be" more valuable to you. Cultivating this feeling of what "makes sense" is one of the most important parts of your education.

Communicate your results. Explain them to others. Don't fear they will catch you in mistakes. There are no silly mistakes, but it's silly to try to hide mistakes you do make. Or, to hide your accomplishments because others might find a mistake and recognize that you are fallible like all other members of the human race. Conversely, it is silly to hide your accomplishments because others might be threatened by your success. If you have to be mediocre to be with your friends, you might want to cultivate some other friends who will support your growth instead of -your mediocrity.

Enjoy your accomplishments!! Acknowledge that YOU DID THAT, YOU SOLVED THAT! Just because work was assigned, or because others are also doing similar or even better work, is no reason to deprecate your own accomplishments. Your own growth is enjoyable, and it can continue for your entire life with no accompanying environmental degradation. HAVE FUN!

Perhaps by now it's obvious that homework problems represent microcosms of virtually ALL real-world problems, from the most trivial to the most significant. Thus, the skills you get in this class (or any class) can have profound value to ALL of your life, especially if you realize what's happening and value the skills for what they are worth.

# **NOTES ON SKILLS**

Analyze your courses, hobbies, jobs, and other experiences in terms of the skills you learn or demonstrate. This will keep you in touch with some of your real motivation for being in those courses, doing those hobbies and jobs. And it will give you some realistic respect for your own abilities. This will stand you in good stead when the time comes for you to create a job or profession for yourself. In many ways, just having a skill is less important than KNOWING you have it, since if you don't know you have it, you won't exercise it or will act without confidence.

Here is a partial list of GENERAL SKILLS, all of which are valuable and worth giving attention and recognition to. Add others of yours that aren't on this list:

CLASSICAL SKILLS:
      Reading with comprehension
      Writing effectively
      Thinking accurately with numbers and algebra

MEMORY

SOCIAL SKILLS
      Cooperating in group or organization
      Supporting others psychologically
      Helping people with problems
      Communicating verbally
      Entertaining
      Persuading
      Stimulating, inspiring
      Supervising, leading
      Doing unpleasant tasks in good spirit
      Being "responsible"
      Dealing sensitively with differing people: education, language, nationality, economic race, behavior, religion, culture, sex, age, background,

PROBLEM-SOLVING SKILLS
      Identifying problem areas; Choosing problems that are personally and socially valuable; Making sense out of new or confused material; Planning; Analyzing and evaluating ideas, alternatives, presentations; Making decisions; Keeping a sense of perspective; Organizing time effectively; Working independently; Working under deadlines; Creativity; Utilizing reference and research materials; Following instructions; Accuracy; Responsiveness; Patience; Working with unpleasant people.